

# Deflections of a Uniformly Loaded Circular Plate With Multiple Support Points

*L.D. Craig*

*Marshall Space Flight Center, Marshall Space Flight Center, Alabama*

*J.A.M. Boulet*

*University of Tennessee, Knoxville, Tennessee*

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National Aeronautics and  
Space Administration

Marshall Space Flight Center • MSFC, Alabama 35812

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## TECHNICAL MEMORANDUM

### DEFLECTIONS OF A UNIFORMLY LOADED CIRCULAR PLATE WITH MULTIPLE SUPPORT POINTS

#### 1. INTRODUCTION

This technical memorandum (TM) describes methods for determining the transverse deflections of a uniformly loaded, thin circular plate of constant thickness supported by single or multiple rings of equally spaced discrete points. The rotations are assumed free at each point. These methods could have application in the design of telescope primary mirror supports that must minimize structural gravitational deformations. They could also be of general use to the structural analyst.

Tables and graphs are presented in section 2 for a variable radius ring of three, four, five, or six equally spaced support points. These contain constants for calculation of the transverse deflection at three locations of interest. Section 3 contains results for multiple rings of various support point configurations. These results include constants for the calculation of root mean square (RMS) and peak-to-valley deflections and the fraction of load supported by each ring. Results obtained from three different methods are summarized and compared. Also presented are equations suitable for programming into a mathematical solver computer program. Once programmed, results may be obtained for practically any support point configuration.

## 2. SINGLE RING OF MULTIPLE SUPPORT POINTS

The series solution for this case is lengthy and will not be shown here, but it can be found in reference 1. The number of support points was varied from three to six and the results presented in tables 1–4. Each table contains the applicable constant used to determine the transverse deflection at a specific location on the plate. The support ring radius is also varied. These data are displayed graphically to better illustrate the results (figs. 1–4). Note that for a support ring radius equal to zero, the result is identical to a uniformly loaded circular plate supported by one point at the center. Note also that for a support ring radius equal to the outer edge radius ( $b/a = 1$ ), the normalized deflection at a support ( $r = a$ ,  $\theta = 0^\circ$ ) is zero, as it should be. As shown in reference 2, there is no significant difference in the results when the number of support points is increased beyond six.

The variables below are defined as follows: transverse deflection  $w$ , uniform load  $q$  in force per unit area, radius  $a$  of the plate, radius  $b$  of the support ring, Poisson's ratio  $\nu$ , and flexural rigidity  $D$ .

Table 1. Normalized deflections for a three-point support ( $\nu=0.25$ ).

$b/a$	$w/(qa^4/D)$		
	On edge $r=a$ , $\theta=0^\circ$	On edge $r=a$ , $\theta=60^\circ$	At center $r=0$ , $\theta=0^\circ$
0.0	0.096875	0.096875	0
0.05	0.09473	0.094743	-0.00079592
0.1	0.090002	0.09011	-0.0023156
0.15	0.083638	0.084002	-0.0040632
0.2	0.076131	0.076988	-0.0057678
0.25	0.067821	0.069488	-0.0072322
0.3	0.058976	0.061836	-0.0082937
0.35	0.049818	0.054326	-0.0088073
0.4	0.04055	0.047218	-0.0086351
0.45	0.031358	0.040755	-0.0076399
0.5	0.022429	0.03517	-0.0056792
0.55	0.013948	0.030692	-0.0026016
0.6	0.0061139	0.027552	-0.0017582
0.65	-0.00086453	0.025985	-0.0075835
0.7	-0.0067566	0.026242	-0.015081
0.75	-0.011304	0.028595	-0.024489
0.8	-0.014209	0.03335	-0.036089
0.85	-0.015115	0.040869	-0.050224
0.9	-0.013569	0.051611	-0.067343
0.95	-0.0089236	0.066226	-0.088087
1.0	0.0	0.085882	-0.11362

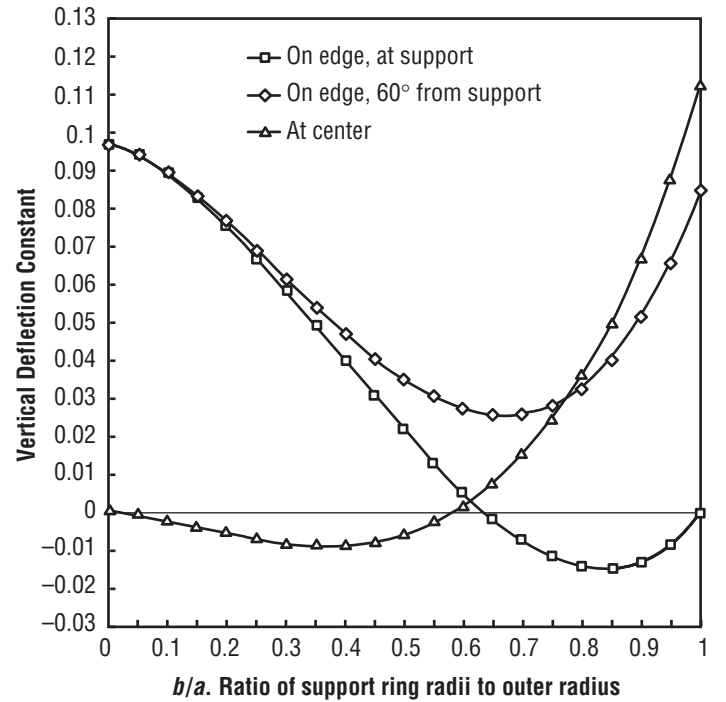


Figure 1. Three-point support.

Table 2. Normalized deflections for a four-point support ( $\nu=0.25$ ).

$b/a$	$w/(qa^4/D)$		
	On edge $r=a, \theta=0^\circ$	On edge $r=a, \theta=45^\circ$	At center $r=0, \theta=0^\circ$
0.0	0.096875	0.096875	0
0.05	0.094718	0.094718	-0.0008143
0.1	0.08998	0.089985	-0.0023891
0.15	0.083642	0.083666	-0.004229
0.2	0.076226	0.076301	-0.0060644
0.25	0.068097	0.068277	-0.0077013
0.3	0.059536	0.059906	-0.0089838
0.35	0.050776	0.051453	-0.009778
0.4	0.042014	0.043154	-0.009963
0.45	0.033426	0.035226	-0.0094251
0.5	0.025173	0.027872	-0.0080536
0.55	0.017411	0.021288	-0.0057361
0.6	0.01029	0.01567	-0.0023553
0.65	0.003966	0.011209	0.0022154
0.7	-0.0013977	0.0081074	0.0081168
0.75	-0.0056214	0.0065752	0.01551
0.8	-0.0085001	0.0068455	0.024585
0.85	-0.0097884	0.0091873	0.035578
0.9	-0.0091717	0.013937	0.0488
0.95	-0.0061952	0.02157	0.064707
1.0	0.0	0.032954	0.084153

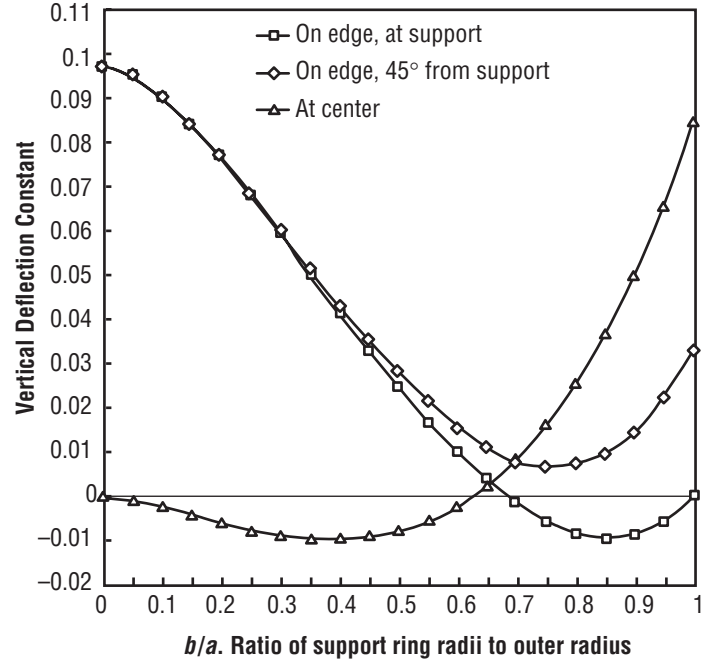


Figure 2. Four-point support.

Table 3. Normalized deflections for a five-point support ( $\nu=0.25$ ).

$b/a$	$w/(qa^4/D)$		
	On edge $r=a, \theta=0^\circ$	On edge $r=a, \theta=36^\circ$	At center $r=0, \theta=0^\circ$
0.0	0.096875	0.096875	0
0.05	0.094712	0.094712	-0.00082045
0.1	0.089958	0.089958	-0.0024137
0.15	0.083598	0.0836	-0.0042844
0.2	0.076161	0.076169	-0.0061629
0.25	0.068021	0.068045	-0.0078555
0.3	0.059469	0.059529	-0.0092066
0.35	0.050746	0.050874	-0.010083
0.4	0.042058	0.042304	-0.010368
0.45	0.033589	0.034023	-0.0099499
0.5	0.0255	0.026222	-0.0087263
0.55	0.017946	0.01908	-0.0065955
0.6	0.011068	0.012775	-0.0034555
0.65	0.0050073	0.0074816	0.00079924
0.7	$-9.4832 \times 10^{-5}$	0.0033756	0.006282
0.75	-0.0040905	0.00063968	0.013118
0.8	-0.0068171	-0.00053068	0.021454
0.85	-0.0080849	$8.4254 \times 10^{-5}$	0.031467
0.9	-0.0076547	0.0027521	0.043391
0.95	-0.0051828	0.0078453	0.057573
1.0	0.0	0.016058	0.074692

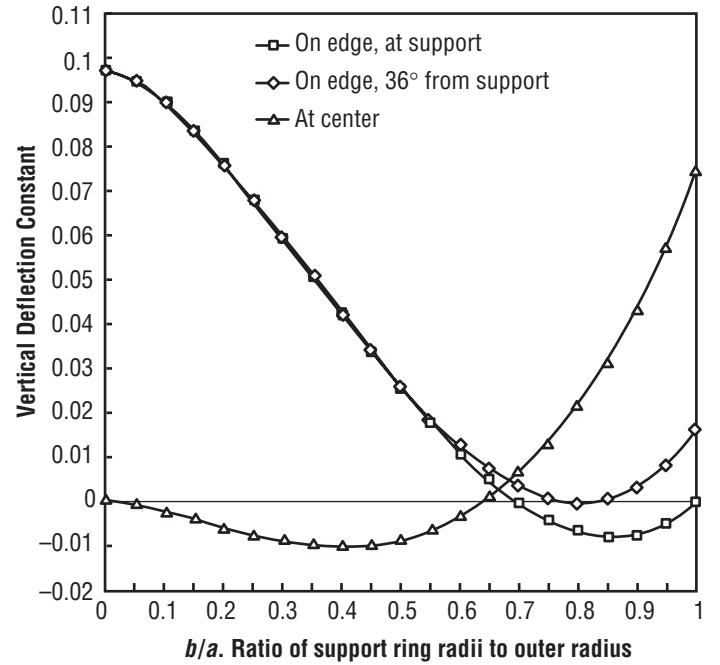


Figure 3. Five-point support.

Table 4. Normalized deflections for a six-point support ( $\nu=0.25$ ).

$b/a$	$w/(qa^4/D)$		
	On edge $r=a, \theta=0^\circ$	On edge $r=a, \theta=30^\circ$	At center $r=0, \theta=0^\circ$
0.0	0.096875	0.096875	0
0.05	0.094709	0.094709	-0.0008231
0.1	0.089947	0.089947	-0.0024243
0.15	0.083575	0.083575	-0.0043082
0.2	0.076122	0.076123	-0.0062054
0.25	0.067965	0.067968	-0.0079218
0.3	0.059398	0.059409	-0.0093022
0.35	0.050666	0.050693	-0.010214
0.4	0.04198	0.042041	-0.010538
0.45	0.033529	0.033649	-0.010168
0.5	0.025479	0.0257	-0.0089995
0.55	0.017987	0.018367	-0.0069353
0.6	0.011196	0.011818	-0.0038785
0.65	0.0052455	0.0062157	0.00026869
0.7	0.00026902	0.0017249	0.0056073
0.75	-0.0035977	-0.0014877	0.012246
0.8	-0.0062118	-0.0032473	0.020307
0.85	-0.0074135	-0.0033622	0.029934
0.9	-0.007007	-0.0016062	0.041316
0.95	-0.0047165	0.0023282	0.054729
1.0	0.0	0.0090156	0.070726

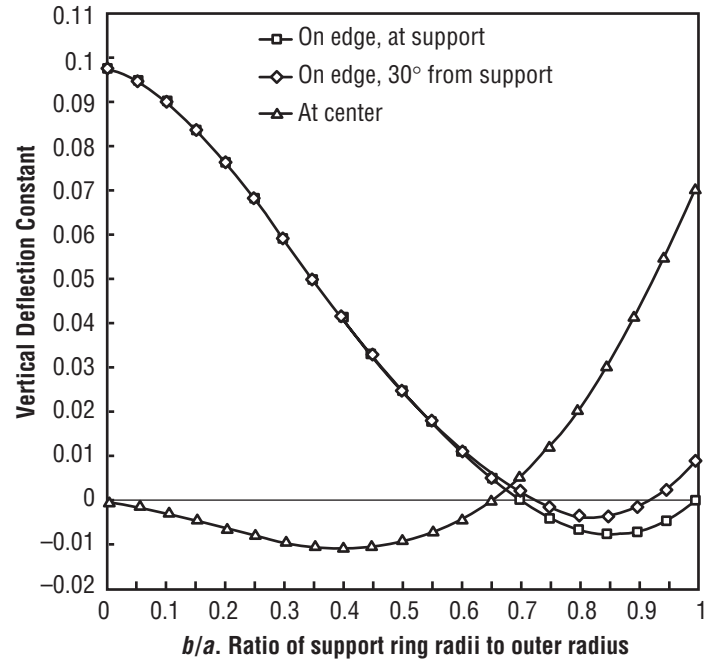


Figure 4. Six-point support.

### 3. MULTIPLE RINGS OF EQUALLY SPACED SUPPORT POINTS

The solution to the multiple ring problem is described in a paper by Nelson, Lubliner, and Mast.<sup>2</sup> A solution for a single ring of discrete support points is derived in appendix B of reference 2. The multiple ring solution<sup>2</sup> is expressed as the summation of single ring solutions with each ring weighted by its portion of the total load reacted. This summation of weighted single ring solutions is not easily obtained for the analyst with only a basic understanding of plate theory and the method of superposition. The solution is complicated and lengthy, but results may be obtained quickly with the aid of a computer.

In 1998, a summer faculty fellow, Dr. Toby Boulet of the University of Tennessee, attempted to develop a true, closed-form solution. In the process, he developed a Mathcad® (a registered trademark of MathSoft, Inc.) document using the solution in reference 2. This document can be used to determine transverse deflections of a uniformly loaded circular plate resting on multiple rings of equally spaced support points, multiple rings of equally spaced support points with a center support point, or a single ring of equally spaced support points with or without a center support. The number of rings and support points must be two or greater to obtain results. Deflections for a single ring of points may be found by specifying different azimuthal positions of two support rings located at the same radius. To create a center support point, the radius of one ring must be set equal to zero. Dr. Boulet programmed the following equations in Mathcad:

$\nu := 0.25$       Poisson's ratio

$$f_1(\xi, \beta) := (\beta^2 + \xi^2) \cdot \ln(\beta) + \frac{1}{2} \cdot \frac{1 - \beta^2}{1 + \nu} \cdot [3 + \nu - (1 - \nu) \cdot \xi^2]$$

$$f_2(\xi, \beta) := (\beta^2 + \xi^2) \cdot \ln(\xi) + \frac{1}{2} \cdot \frac{1 - \xi^2}{1 + \nu} \cdot [3 + \nu - (1 - \nu) \cdot \beta^2]$$

$$f_u(\xi, \beta) := \left( \frac{1 - \xi^2}{8} \right) \cdot \left( \frac{5 + \nu}{1 + \nu} - \xi^2 \right) - \text{if}(\xi < \beta, f_1(\xi, \beta), f_2(\xi, \beta))$$

$J := 20$  (number of terms in Fourier expansion)

$J := 1, 2, \dots, J$

$$A(j, \beta, N) := \frac{\beta^{N \cdot j}}{3 + \nu} \cdot \left[ (1 - \nu) \cdot \left( \frac{1}{N \cdot j - 1} - \frac{\beta^2}{N \cdot j} \right) + \frac{8}{(N \cdot j)^2} \cdot \frac{1 + \nu}{(N \cdot j - 1) \cdot (1 - \nu)} \right]$$

$$B(j, \beta, N) := -\beta^{N \cdot j} \cdot \frac{1 - \nu}{3 + \nu} \cdot \left( \frac{1}{N \cdot j} - \frac{\beta^2}{N \cdot j + 1} \right)$$

$$C(j, \beta, N) := \frac{-1 \cdot \beta^{N \cdot j + 2}}{N \cdot j \cdot (N \cdot j + 1)}$$

$$D(j, \beta, N) := \frac{\beta^{N \cdot j}}{N \cdot j \cdot (N \cdot j - 1)}$$

$$E(j, \beta, N) := A(j, \beta, N) + \frac{\beta^{-(N \cdot j) + 2}}{N \cdot j \cdot (N \cdot j - 1)}$$

$$F(j, \beta, N) := B(j, \beta, N) - \frac{\beta^{-(N \cdot j)}}{N \cdot j \cdot (N \cdot j + 1)}$$

$$\eta(j, \xi, \beta, N) := A(j, \beta, N) \cdot \xi^{N \cdot j} + B(j, \beta, N) \cdot \xi^{N \cdot j + 2} + C(j, \beta, N) \cdot \xi^{-(N \cdot j)} + D(j, \beta, N) \cdot \xi^{-(N \cdot j) + 2}$$

$$\lambda(j, \xi, \beta, N) := E(j, \beta, N) \cdot \xi^{N \cdot j} + F(j, \beta, N) \cdot \xi^{N \cdot j + 2}$$

$$w_j(j, \xi, \beta, N) := \text{if}(\xi < \beta, \lambda(j, \xi, \beta, N), \eta(j, \xi, \beta, N))$$

$$F(\xi, \beta, N, \theta) := f_u(\xi, \beta) - \left( \sum_j w_j(j, \xi, \beta, N) \cdot \cos(N \cdot j \cdot \theta) \right)$$

$$N_R := ? \quad (\text{integer number of rings})$$

$$N := \begin{bmatrix} N_1 \\ N_2 \\ \vdots \\ \vdots \\ N_{N_R} \end{bmatrix} \quad N_i \text{ is the number of supports in each ring } (>1).$$

$$\beta := \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_{N_R} \end{bmatrix} \quad \beta_i \text{ is the ring radius divided by the outer radius of the plate.}$$

$$\phi := \begin{bmatrix} \phi_1 \\ \phi_2 \\ \vdots \\ \phi_{N_R} \end{bmatrix} \quad \phi_i \text{ is the azimuthal (clocking angle) location (radians) of supports in each ring.}$$

Note that the supports within each ring are equally spaced; that is, if  $N_1=3$ , the support points in ring 1 are  $120^\circ$  apart or, if  $N_1=4$ , they are  $90^\circ$  apart and so on.

$$s := 1, 2, \dots, N_R \qquad k := 1, 2, \dots, N_R$$

$$b_{k,s} := F(\beta_k, \beta_s, N_s, \phi_k - \phi_s)$$

$$t := 2, 3, \dots, N_R$$

$$K_{1,k} := 1$$

$$K_{t,k} := b_{t,k} - b_{1,k}$$

$$h := XX.XX \cdot \text{in.}$$

$$p := YY.YY \cdot \frac{\text{lbf}}{\text{in.}^3} \cdot h$$

$$c_1 := p$$

$$c_t := 0 \cdot \text{psi}$$

$$q := K^{-1} \cdot c$$

$$E_{\text{modulus}} := ZZ.ZZ \cdot \text{psi}$$

$$\Delta := \frac{E_{\text{modulus}} \cdot h^3}{12 \cdot (1 - \nu^2)} \quad \frac{q}{p} = \left[ \begin{array}{c} \\ \\ \end{array} \right]$$

$$a := RR.RR \cdot \text{in.}$$

$$w(\xi, \theta) := \frac{a^4}{8 \cdot \Delta} \cdot \left[ \sum_k q_k \cdot (F(\xi, \beta_k, N_k, \theta - \phi_k) - F(\beta_1, \beta_k, N_k, -\phi_k)) \right]$$

$$N_n := nnn$$

$$N_{\kappa} := kkk$$

where  $N_n$  is the number of circumferential points at which the deflection is calculated and  $N_{\kappa}$  is the number of radial points.

$$n := 1, 2, \dots N_n$$

$$k := 1, 2, \dots N_{\kappa}$$

$$r_{\kappa} := \kappa \cdot \frac{a}{N_{\kappa}}$$

$$\theta_n := -\pi + (n - 1) \cdot \frac{2 \cdot \pi}{N_n}$$

$$d_{\kappa,n} := w\left(\frac{r_{\kappa}}{a}, \theta_n\right) \quad \text{transverse displacement at } r, \theta$$

$$PV := \max(d) - \min(d)$$

$$PV = \quad \max(d) =$$

$$\min(d) =$$

The following equations calculate the Zernike coefficients for bias,  $x$  tilt, and  $y$  tilt:

$$C_0 := \frac{1}{\pi} \cdot \int_{-\pi}^{\pi} \int_0^1 w(\xi, \theta) \cdot \xi d\xi d\theta \quad C_0 =$$

$$C_1 := \frac{4}{\pi \cdot a} \cdot \int_{-\pi}^{\pi} \int_0^1 w(\xi, \theta) \cdot \xi^2 d\xi \cdot \cos(\theta) d\theta \quad C_1 =$$

$$C_2 := \frac{4}{\pi \cdot a} \cdot \int_{-\pi}^{\pi} \int_0^1 w(\xi, \theta) \cdot \xi^2 d\xi \cdot \sin(\theta) d\theta \quad C_2 =$$

The following equations remove the bias from the transverse deflections and calculate the residual RMS deflection:

$$\delta(\xi, \theta) := w(\xi, \theta) - C_0$$

$$\delta_{\text{RMS}} := \sqrt{\frac{1}{\pi} \cdot \int_{-\pi}^{\pi} \int_0^1 \delta(\xi, \theta)^2 \cdot \xi d\xi d\theta} \quad \delta_{\text{RMS}} =$$

Calculate  $\gamma$  for comparison with reference 2.

$$SA := \pi \cdot a^2 \quad N_S := \sum N \quad (\text{sum of the support points})$$

$$\gamma_N := \delta_{\text{RMS}} \cdot \frac{\Delta}{p} \cdot \left(\frac{N_S}{SA}\right)^2 \quad \text{reference 2, equation 4 solved for } \gamma_N.$$

$$\gamma_N =$$

This concludes the Mathcad input. The cases in table 1 of reference 2 were solved with this input and the results converted to a form for comparison. The same cases were solved via the finite element method (FEM) with NASTRAN for further validation, and these results are also shown in table 5. No attempt was made to explain the discrepancies between the results from reference 2 and those from NASTRAN or the equations above. To determine the RMS deflection (with bias removed; that is, the first Zernike coefficient), use equation (4) from reference 2 shown below.

$$\delta_{\text{RMS}} = \gamma_N \cdot \frac{q}{D} \cdot \left( \frac{SA}{N_S} \right)^2 .$$

Mathcad or NASTRAN deflections may be illustrated graphically with various software packages. Once the results are generated, they can be plotted internally or exported to a spreadsheet and plotted externally. Results for the 4-ring, 12-point support in table 5 are shown in figures 5–9. Note that deflection downward is positive with the exception of the PATRAN fringe plot of NASTRAN results where the downward direction is negative.

The Mathcad-generated deadweight deflections shown in figure 5 are for a 0.1-in. thick, 20-in. diameter aluminum plate. Note the zero deflection at the inner and intermediate ring support points. Figures 6–9 illustrate different ways of displaying the deadweight deflections of the aluminum plate.

Table 5. Deflection constants and reactions for various multipoint support configurations ( $\nu=0.25$ ).

$N_s$	$\beta$	$\phi$ deg.	$\gamma_N \times 10^3$ (NLM)	$\gamma_N \times 10^3$ (FEM)	$\gamma_N \times 10^3$ (Boulet)	$P-V/RMS$ (NLM)	$P-V/RMS$ (FEM)	$P-V/RMS$ (Boulet)	$\varepsilon$ (NLM)	$\varepsilon$ (FEM)	$\varepsilon$ (Boulet)
3	0.645		5.76	5.73	5.76	4.2	4.19	4.19	1.0	1.0	1.0
6	0.681		2.93	2.91	2.90	4.3	4.31	4.31	1.0	1.0	1.0
7	0.0 0.737		2.36	3.00	2.93	4.9	4.81	4.88	0.1183 0.8817	0.1301 0.8686	0.1301 0.87
9	0.2825 0.7936 0.770	0.0 0.0 60.0	3.76	4.83	4.75	5.0	4.42	4.44	0.2309 0.3637 0.4054	0.2365 0.3573 0.3962	0.2450 0.3582 0.3968
12	0.3151 0.7662 0.8257 0.8257	0.0 60.0 20.0 -20.0	1.94	2.13	2.07	5.1	5.01	5.0	0.2783 0.2843 0.2187 0.2187	0.2781 0.2804 0.2201 0.2201	0.2786 0.2805 0.2204 0.2204
15	0.3192 0.7765 0.8412 0.7765 0.8412	0.0 44.88 15.0 -44.88 -15.0	2.32	3.00	2.97	5.4	4.18	4.16	0.2833 0.2046 0.1538 0.2046 0.1538	0.2810 0.2030 0.1565 0.2030 0.1565	0.2810 0.2037 0.1558 0.2037 0.1558
18	0.4741 0.3195 0.8171 0.8536 0.8171 0.8536	0.0 60.0 44.8 15.26 -44.8 -15.26	1.89	2.09	2.02	5.5	4.89	5.01	0.1625 0.2071 0.1731 0.1421 0.1731 0.1421	0.1689 0.2018 0.1731 0.1416 0.1731 0.1416	0.1704 0.2008 0.1730 0.1414 0.1730 0.1414
36	0.2569 0.5771 0.5771 0.8830 0.8834 0.8830	0.0 15.18 44.82 9.76 30.0 50.24	1.63	1.71	1.65	6.0	5.27	5.38	0.1671 0.1812 0.1812 0.1549 0.1607 0.1549	0.1687 0.1791 0.1791 0.1571 0.1577 0.1571	0.1674 0.1810 0.1810 0.1552 0.1602 0.1552

$\beta$  is the ratio of the ring radius to the outer radius

$\phi$  is clocking or the azimuthal location (in degrees) of supports in one ring relative to one of the other rings

$\varepsilon$  is the fraction of load carried by one ring of supports, equal to  $q/p$  above

NLM is Nelson, Lubliner, and Mast.<sup>2</sup>

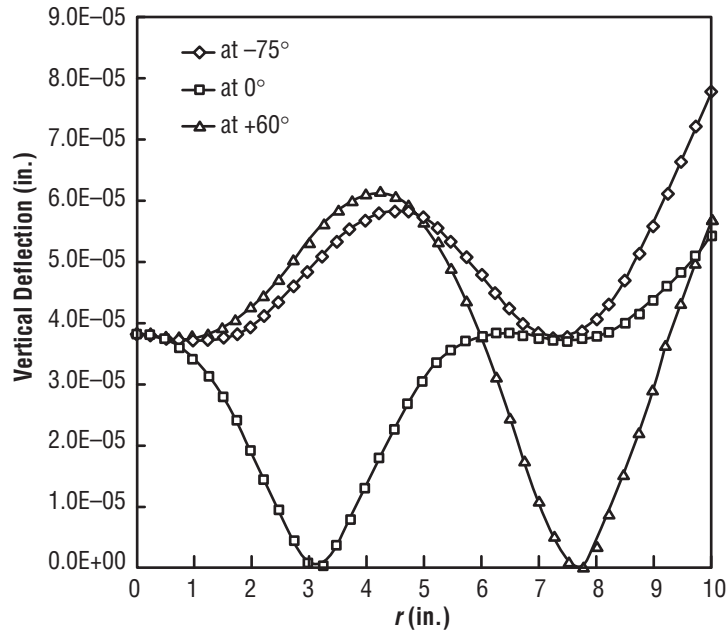


Figure 5. Deflection versus  $r$  at three azimuthal locations.

Figures 6 and 7 are three-dimensional surface plots of the transverse deflections and display the deformed shape. Figures 8 and 9 show the fringe plots of the NASTRAN and Mathcad results.

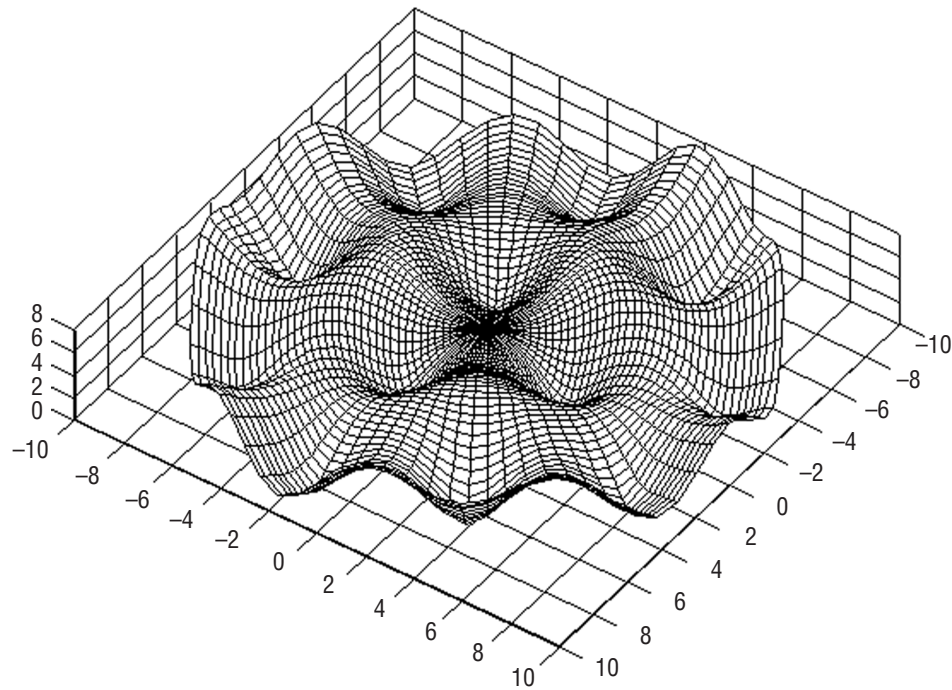


Figure 6. Mathcad surface plot of Mathcad results.

MSC/PATRAN Version 8.022-Feb-99 13:15:49  
Deform: -1GZ on 12 PTS.SC1, Static Subcase; Displacements, Translational

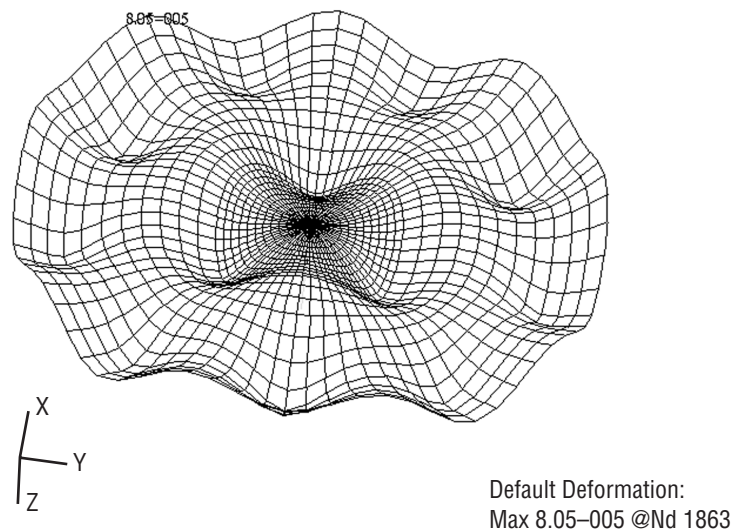


Figure 7. PATRAN surface plot of NASTRAN results.

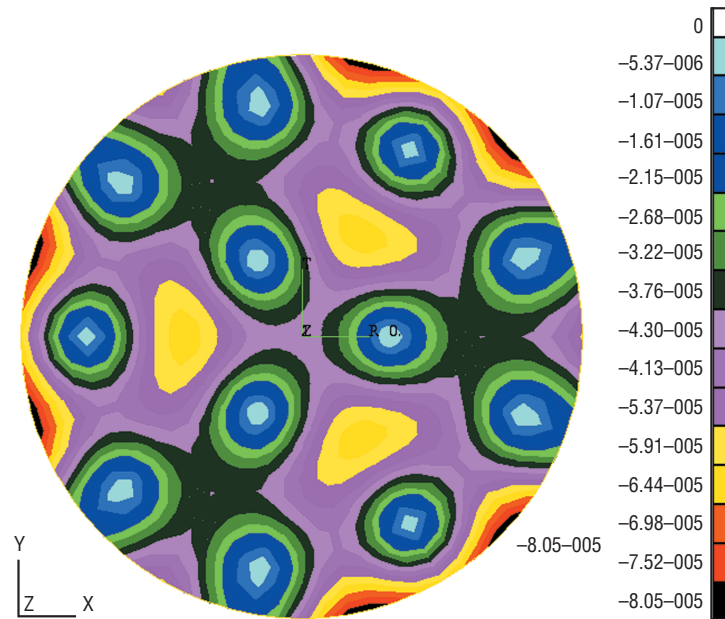


Figure 8. PATRAN fringe plot of NASTRAN results.

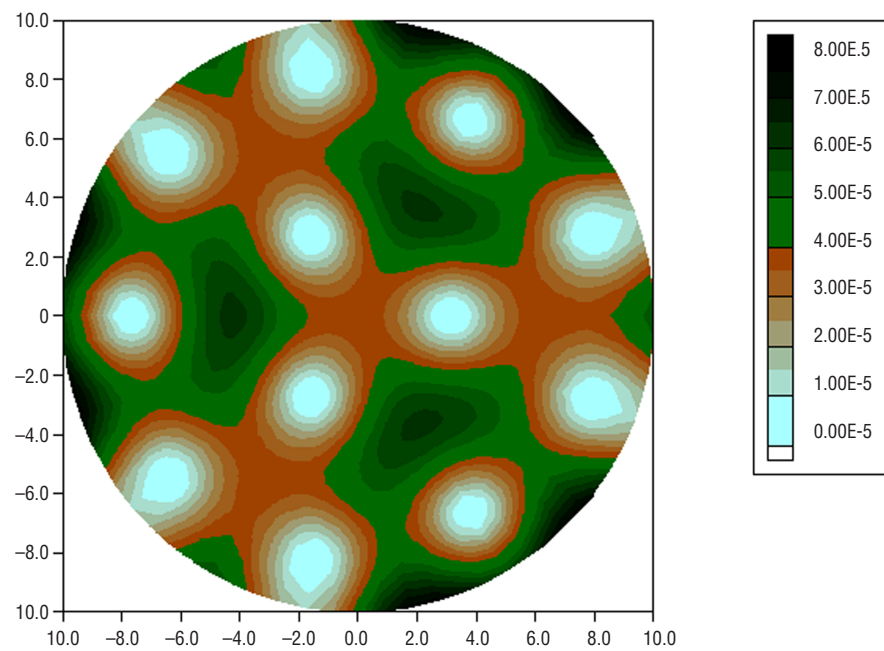


Figure 9. Fringe plot of Mathcad results.

#### 4. CONCLUSIONS

This TM describes three methods for defining the deflected shape of a uniformly loaded, thin circular flat plate supported with multiple discrete points. A comparison of these methods for specific examples is shown. These methods can provide a preliminary support system design for thin telescope mirrors. The equations programmed into Mathcad can solve virtually any system of support points but are limited to thin, circular flat plates (although contribution to the deflection due to shear could be added). The finite element method (NASTRAN) can solve any support system and mirror geometry but requires much more computer time and memory and more of the analyst's time and effort. The tables and graphs contained in section 2 are subsets of the results of section 3 and are generated with different equations. The graphs may be used to determine the optimum support ring radius.

## REFERENCES

1. Pan, H.H.; and Yu, J.C.L.: “Uniformly Loaded Circular Plate Supported at Discrete Points,” *International Journal of Mechanical Sciences*, pp. 333–340, May 1966.
2. Nelson, J.E.; Lubliner, J.; and Mast, T.S.: “Telescope Mirror Supports: Plate Deflections on Point Supports,” *SPIE*, Vol. 332, pp. 212–228, 1982.

<b>REPORT DOCUMENTATION PAGE</b>			Form Approved OMB No. 0704-0188	
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13. ABSTRACT (Maximum 200 words)  This technical memorandum describes a method for determining the transverse deflections of a uniformly loaded, thin circular plate of constant thickness supported by single or multiple rings of equally spaced discrete points. The rotations are assumed free at each point. This could have application in the design of telescope mirror supports that must minimize structural gravitational deformations. It could also be of general use to the structural analyst.				
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